The hydrogen atom revisited

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Abstract

Several myths about quantum mechanics exist due to a loss of awareness of its details since its inception in the beginning of the last century or based on recent experimental evidence. It is taught in textbooks that atomic hydrogen cannot go below the ground state. Atomic hydrogen having an experimental ground state of 13.6 eV can only exist in a vacuum or in isolation, and atomic hydrogen cannot go below this ground state in isolation. However, there is no known composition of matter containing hydrogen in the ground state of 13.6 eV. It is a myth that hydrogen has a theoretical ground state based on first principles. Historically, there were many directions in which to proceed to solve a wave equation for hydrogen. The Schrodinger equation gives the observed spontaneously radiative states and the nonradiative energy level of atomic hydrogen. On this basis alone, it is justified despite its inconsistency with physical laws as well as with many experiments. A solution compatible with first principles and having first principles as the basis of quantization was never found. Scattering results required the solution to be interpreted as probability waves that give rise to the uncertainty principle which in turn forms the basis of the wave particle duality. The correspondence principal predicts that quantum predictions must approach classical predictions on a large scale. However, recent data has shown that the Heisenberg uncertainty principle as the basis of the wave particle duality and the correspondence principle taught in textbooks are experimentally incorrect. Recently, a reconsideration of the postulates of quantum mechanics, has given rise to a closed form solution of a Schrodinger-like wave equation based on first principles. Hydrogen at predicted lower energy levels has been identified in the extreme ultraviolet emission spectrum from interstellar medium. In addition, new compositions of matter containing hydrogen at predicted lower energy levels have recently been observed in the laboratory, which energy levels are achieved using the novel catalysts. © 2000 International Association for Hydrogen Energy. Published by Elsevier Science Ltd. All rights reserved.

1. Introduction

J.J. Balmer showed, in 1885, that the frequencies for some of the lines observed in the emission spectrum of atomic hydrogen could be expressed with a completely empirical relationship. This approach was later extended by J.R. Rydberg, who showed that all of the spectral lines of atomic hydrogen were given by the equation

$$\bar{\nu} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right), \tag{1}$$

where $R = 109,677$ cm^{-1}, $n_f = 1, 2, 3, \ldots, n_i = 2, 3, 4, \ldots,$ and $n_i > n_f$. Niels Bohr, in 1913, developed a theory for atomic hydrogen based on an unprecedented postulate of

stable circular orbits that do not radiate. Although no explanation was offered for the existence of stability for these orbits, the results gave energy levels in agreement with Rydberg's equation

$$E_n = -\frac{e^2}{n^2 8 \pi \varepsilon_0 a_H} = -\frac{13.598 \text{ eV}}{n^2}, \tag{2}$$

$$n = 1, 2, 3, \ldots, \tag{3}$$

where a_H is the Bohr radius for the hydrogen atom (52.947 pm), e is the magnitude of the charge of the electron, and ε_0 is the vacuum permittivity. Bohr's theory was a straightforward application of Newton's laws of motion and Coulomb's law of electric force — both pillars of classical physics and is in accord with the experimental observation that atoms are stable. However, it is not in accord with electromagnetic theory — another pillar of classical physics which predicts that accelerated charges radiate energy in the form

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of electromagnetic waves. An electron pursuing a curved path is accelerated and therefore should continuously lose energy, spiraling into the nucleus in a fraction of a second. The predictions of electromagnetic theory have always agreed with experiment, yet atoms do not collapse. To the early 20th century theoreticians, this contradiction could mean only one thing: The laws of physics that are valid in the macroworld do not hold true in the microworld of the atom. In 1923, de Broglie suggested that the motion of an electron has a wave aspect — $\lambda = h/p$. This concept seemed unlikely according to the familiar properties of electrons such as charge, mass and adherence to the laws of particle mechanics. But, the wave nature of the electron was confirmed by Davisson and Germer in 1927 by observing diffraction effects when electrons were reflected from metals. Schrödinger reasoned that if electrons have wave properties, there must be a wave equation that governs their motion. And in 1926, he proposed the Schrödinger equation, $H\Psi = E\Psi$, where Ψ is the wave function, H is the wave operator, and E is the energy of the wave. This equation, and its associated postulates, is now the basis of *quantum mechanics*, and it is the basis for the world view that the atomic realm including the electron and photon cannot be described in terms of "pure" wave and "pure" particle but in terms of a wave-particle duality. The wave-particle duality based on the fundamental principle that physics on an atomic scale is very different from physics on a macroscopic scale is central to present day atomic theory [1].

2. Development of atomic theory

2.1. Bohr theory

In 1911, Rutherford proposed a planetary model for the atom where the electrons revolved about the nucleus (which contained the protons) in various orbits to explain the spectral lines of atomic hydrogen. There was, however, a fundamental conflict with this model and the prevailing classical physics. According to classical electromagnetic theory, an accelerated particle radiates energy (as electromagnetic waves). Thus, an electron in a Rutherford orbit, circulating at constant speed but with a continually changing direction of its velocity vector is being accelerated; thus, the electron should constantly lose energy by radiating and spiral into the nucleus.

An explanation was provided by Bohr in 1913, when he assumed that the energy levels were quantized and the electron was constrained to move in only one of a number of allowed states. Niels Bohr's theory for atomic hydrogen was based on an unprecedented postulate of stable circular orbits that do not radiate. Although no explanation was offered for the existence of stability for these orbits, the results gave energy levels in agreement with Rydberg's equation. Bohr's theory was a straightforward application of Newton's laws

of motion and Coulomb's law of electric force. According to Bohr's model, the point particle electron was held to a circular orbit about the relatively massive point particle nucleus by the balance between the coulombic force of attraction between the proton and the electron and centrifugal force of the electron.

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}. \tag{4}$$

Bohr postulated the existence of stable orbits in defiance of classical physics (Maxwell's equations), but he applied classical physics according to Eq. (4). Then Bohr realized that the energy formula (Eqs. (2) and (3)) was given by postulating nonradiative states with angular momentum

$$L_z = m_e v r = n\hbar, \quad n = 1, 2, 3 \ldots. \tag{5}$$

and by solving the energy equation classically. The Bohr radius is given by substituting the solution of Eq. (5) for v into Eq. (4).

$$r = \frac{4\pi\varepsilon_0 \hbar^2 n^2}{m_e e^2} = n^2 a_0, \quad n = 1, 2, 3 \ldots. \tag{6}$$

The total energy is the sum of the potential energy and the kinetic energy. In the present case of an inverse squared central field, the total energy (which is the negative of the binding energy) is one-half the potential energy [2]. The potential energy, $\phi(r)$, is given by Poisson's equation

$$\phi(\mathbf{r}) = -\int_{V'} \frac{\rho(\mathbf{r}') \, dv'}{4\pi\varepsilon_0 |\mathbf{r} - \mathbf{r}'|}. \tag{7}$$

For a point charge at a distance r from the nucleus the potential is

$$\phi(r) = -\frac{e^2}{4\pi\varepsilon_0 r}. \tag{8}$$

Thus, the total energy is given by

$$E = -\frac{Z^2 e^2}{8\pi\varepsilon_0 r}, \tag{9}$$

where $Z = 1$. Substitution of Eq. (6) into Eq. (9) with the replacement of the electron mass by the reduced electron mass gives Eqs. (2) and (3).

Bohr's model was in agreement with the observed hydrogen spectrum, but it failed with the helium spectrum, and it could not account for chemical bonds in molecules. The prevailing wisdom was that the Bohr model failed because it was based on the application of Newtonian mechanics for discrete particles. Its limited applicability was attributed to the unwarranted assumption that the energy levels are quantized.

Bohr's theory may also be analyzed according to the corresponding energy equation. Newton's differential equations of motion in the case of the central field such as a gravitational or electrostatic field are

$$m(\ddot{r} - r\dot{\theta}^2) = f(r), \tag{10}$$

$$m(2\dot{r}\dot{\theta} + r\ddot{\theta}) = 0, \tag{11}$$

where $f(r)$ is the central force. The second or transverse equation, Eq. (11), gives the result that the angular momentum is constant.

$$r^2\dot{\theta} = \text{constant} = L/m, \tag{12}$$

where L is the angular momentum. The central force equations can be transformed into an orbital equation by the substitution, $u = 1/r$. The differential equation of the orbit of a particle moving under a central force is

$$\frac{\delta^2 u}{\delta\theta^2} + u = \frac{-1}{mL^2u^2/m^2} f(u^{-1}). \tag{13}$$

Because the angular momentum is constant, motion in only one plane need be considered; thus, the orbital equation is given in polar coordinates. The solution of Eq. (13) for an inverse square force

$$f(r) = -\frac{k}{r^2} \tag{14}$$

is

$$r = r_0 \frac{1 + e}{1 + e\cos\theta}, \tag{15}$$

$$e = A\frac{mL^2/m^2}{k}, \tag{16}$$

$$r_0 = \frac{mL^2/m^2}{k(1 + e)}, \tag{17}$$

where e is the eccentricity and A is a constant. The equation of motion due to a central force can also be expressed in terms of the energies of the orbit. The square of the speed in polar coordinates is

$$v^2 = (\dot{r}^2 + r^2\dot{\theta}^2). \tag{18}$$

Since a central force is conservative, the total energy, E, is equal to the sum of the kinetic, T, and the potential, V, and is constant. The total energy is

$$\frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) + V(r) = E = \text{constant}. \tag{19}$$

Substitution of the variable $u = 1/r$ and Eq. (12) into Eq. (19) gives the orbital energy equation

$$\frac{1}{2}m\frac{L^2}{m^2}\left[\left(\frac{\delta^2 u}{\delta\theta^2}\right) + u^2\right] + V(u^{-1}) = E. \tag{20}$$

Because the potential energy function $V(r)$ for an inverse square force field is

$$V(r) = -\frac{k}{r} = -ku, \tag{21}$$

the energy equation of the orbit (Eq. (20)), is

$$\frac{1}{2}m\frac{L^2}{m^2}\left[\left(\frac{\delta^2 u}{\delta\theta^2}\right) + u^2\right] - ku = E, \tag{22}$$

$$\left[\frac{\delta^2 u}{\delta\theta^2} + u^2\right] - \frac{2m}{L^2}[E + ku] = 0, \tag{23}$$

which has the solution

$$r = \frac{m(L^2/m^2)k^{-1}}{1 + [1 + 2Em(L^2/m^2)k^{-2}]^{1/2}\cos\theta}, \tag{24}$$

where the eccentricity, e is

$$e = \left[1 + 2Em\frac{L^2}{m^2}k^{-2}\right]^{1/2}. \tag{25}$$

Eq. (25) permits the classification of the orbits according to the total energy, E, as follows:

$E < 0,\quad e < 1,\quad$ ellipse,

$E < 0,\quad e = 0,\quad$ circle (special case of ellipse),

$E = 0,\quad e = 1,\quad$ parabolic orbit,

$E > 0,\quad e > 1,\quad$ hyperbolic orbit. $\tag{26}$

Since $E = T + V$ and is constant, the closed orbits are those for which $T < |V|$, and the open orbits are those for which $T \geqslant |V|$. It can be shown that the time average of the kinetic energy, $\langle T \rangle$, for elliptic motion in an inverse square field is $1/2$ that of the time average of the potential energy, $\langle V \rangle$. $\langle T \rangle = 1/2\langle V \rangle$.

Bohr's solution is trivial in that he specified a circular bound orbit which determined that the eccentricity was zero, and he specified the angular momentum as a integer multiple of Planck's constant bar. Eq. (25) in CGS units becomes

$$E = -\frac{1}{2}\frac{me^4}{n^2\hbar^2} = -\frac{e^2}{2n^2a_0}. \tag{27}$$

2.2. Schrödinger theory of the hydrogen atom

In 1923, de Broglie suggested that the motion of an electron has a wave aspect — $\lambda = h/p$. This was confirmed by Davisson and Germer in 1927 by observing diffraction effects when electrons were reflected from metals. Schrödinger reasoned that if electrons have wave properties, there must be a wave equation that governs their motion. And, in 1926, he proposed the time-independent Schrödinger equation

$$H\Psi = E\Psi, \tag{28}$$

where Ψ is the wave function, H is the wave operator, and E is the energy of the wave. To give the sought three quantum numbers, the Schrödinger equation solutions are three dimensional in space and four dimensional in spacetime

$$\left[\nabla^2 - \frac{1}{v^2} \frac{\delta^2}{\delta t^2} \right] \Psi(r, \theta, \phi, t) = 0, \tag{29}$$

where $\Psi(r, \theta, \phi, t)$ according to quantum theory is the probability density function of the electron as described below. When the time harmonic function is eliminated [3]

$$-\frac{\hbar^2}{2\mu} \left[\frac{1}{r^2} \frac{\delta}{\delta r} \left(r^2 \frac{\delta \Psi}{\delta r} \right) + \frac{1}{r^2 \sin \theta} \frac{\delta}{\delta \theta} \left(\sin \theta \frac{\delta \Psi}{\delta \theta} \right)_{r,\phi} \right.$$
$$\left. + \frac{1}{r^2 \sin^2 \theta} \left(\frac{\delta^2 \Psi}{\delta \phi^2} \right)_{r,\theta} \right] + V(r) \Psi(r, \theta, \phi) = E \Psi(r, \theta, \phi), \tag{30}$$

where the potential energy $V(r)$ in CGS units is

$$V(r) = -\frac{e^2}{r}. \tag{31}$$

The Schrödinger equation (Eq. (30)) can be transformed into a sum comprising a part that depends only on the radius and a part that is a function of angle only obtained by separation of variables and linear superposition in spherical coordinates. The general form of the solutions for $\psi(r, \theta, \phi)$ are

$$\psi(r, \theta, \phi) = \sum_{l,m} R_{nlm}(r) Y_{lm}(\theta, \phi), \tag{32}$$

where l and m are separation constants. The azimuthal (θ) part of Eq. (30) is the generalized Legendre equation which is derived from the Laplace equation by Jackson (Eq. (3.9) of Jackson [4]). The solutions for the full angular part of Eq. (30), $Y_{lm}(\theta, \phi)$, are the spherical harmonics

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)(l-m)!}{4\pi(l+m)!}} P_l^m(\cos \theta) e^{im\phi}. \tag{33}$$

By substitution of the eigenvalues corresponding to the angular part [5], the Schrödinger equation becomes the radial equation, $R(r)$, given by

$$-\frac{\hbar^2}{2mr^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \left[\frac{\hbar^2 l(l+1)}{2mr^2} + V(r) \right] R(r) = ER(r). \tag{34}$$

The time-independent Schrödinger equation is similar to Eq. (20) except that the solution is for the distribution of a spatial wavefunction in three dimensions rather than the dynamical motion of a point particle of mass m along a one-dimensional trajectory. Electron motion is implicit in the Schrödinger equation. For wave propagation in three dimensions, the full time-dependent Schrödinger equation is required, whereas the classical case contains time

derivatives. The kinetic energy of rotation is K_{rot} is given classically by

$$K_{rot} = \tfrac{1}{2} m r^2 \omega^2, \tag{35}$$

where m is the mass of the electron. In the time-independent Schrödinger equation, the kinetic energy of rotation K_{rot} is given by

$$K_{rot} = \frac{\ell(\ell+1)\hbar^2}{2mr^2}, \tag{36}$$

where

$$L = \sqrt{\ell(\ell+1)\hbar^2} \tag{37}$$

is the magnitude of the electron angular momentum L for the state $Y_{lm}(\theta, \phi)$.

In the case of the ground state of hydrogen, the Schrödinger equation solution is trivial for an implicit circular bound orbit which determines that the eccentricity is zero, and with the specification that the electron angular momentum is Planck's constant bar. With $k = e^2$, Eq. (25) in CGS units becomes

$$E = -\frac{1}{2} \frac{me^4}{\hbar^2} = -\frac{e^2}{2a_0}, \tag{38}$$

which corresponds to $n = 1$ in Eq. (27). Many problems in classical physics give three quantum numbers when three spatial dimensions are considered. In order to obtain three quantum numbers, the Schrödinger equation requires that the solution is for the distribution of a spatial wavefunction in three dimensions with implicit motion rather than a one dimensional trajectory of a point particle as shown below. However, this approach gives rise to predictions about the angular momentum and angular energy which are not consistent with experimental observations as well as a host of other problems which are summarized in Section 10.

The radial equation may be written as

$$\frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) + \frac{2mr^2}{\hbar^2} \left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2} \right] R(r) = 0. \tag{39}$$

Let $U(r) = rR(r)$, then the radial equation reduces to

$$U'' + \frac{2m}{\hbar^2} \left[E - V(r) - \frac{l(l+1)\hbar^2}{2mr^2} \right] U = 0, \tag{40}$$

where

$$\psi = \frac{1}{r} U_{lm}(r) Y_{lm}(\theta, \phi). \tag{41}$$

Substitution of the potential energy given by Eq. (31) into Eq. (40) gives for sufficiently large r

$$U''_\infty - \left(\frac{\alpha}{2} \right)^2 U = 0 \tag{42}$$

provided we define

$$\left(\frac{\alpha}{2} \right)^2 = \frac{-2mE}{\hbar^2}, \tag{43}$$

where α is the eigenvalue of the eigenfunction solution of the Schrödinger equation given *infra* having units of reciprocal length and E is the energy levels of the hydrogen atom. To arrive at the solution which represents the electron, a suitable boundary condition must be imposed. Schrödinger postulated a boundary condition: $\Psi \to 0$ as $r \to \infty$, which leads to a purely mathematical model of the electron. This equation is not based on first principles, has no validity as such, and should not be represented as so. The right-hand side of Eq. (43) must be *postulated* in order that the Rydberg equation is obtained as shown below. The postulate is implicit since Eq. (43) arises from the Schrödinger equation which is postulated. It could be defined *arbitrarily*, but is justified because it gives the Rydberg formula. That Schrödinger guessed the accepted approach is not surprising since many approaches were contemplated at this time [6], and since none of these approaches were superior, Schrödinger's approach prevailed.

The solution of Eq. (42) that is consistent with the boundary condition is

$$U_\infty = c_1 e^{(\alpha/2)r} + c_2 e^{-(\alpha/2)r}. \tag{44}$$

In the case that α is real, the energy of the particle is negative. In this case U_∞ will not have an integrable square if c_1 fails to vanish wherein the radial integral has the form

$$\int_0^\infty R^2 r^2 \, dr = \int U_\infty^2 \, dr. \tag{45}$$

It is shown below that the solution of the Schrödinger corresponds to the case wherein c_1 fails to vanish. Thus, the solutions with sufficiently large r are infinite. The same problem arises in the case of a free electron that is ionized from hydrogen. If α is imaginary, which means that E is positive, Eq. (42) is the equation of a linear harmonic oscillator [7]. U_∞ shows sinusoidal behavior; thus, the wavefunction for the free electron cannot be normalized and is infinite. In addition, the angular momentum of the free electron is infinite since it is given by $\ell(\ell + 1)\hbar^2$ (Eq. (37)) where $\ell \to \infty$.

In order to solve the bound electron states, let

$$E = -W \tag{46}$$

so that W is positive. In Eq. (39), let $r = x/\alpha$ where α is given by Eq. (43)

$$x\frac{d^2R}{dx^2} + 2\frac{dR}{dx} + \left[\frac{2me^2}{\hbar^2\alpha} - \frac{x}{4} - \frac{l(l+1)}{x}\right]R = 0. \tag{47}$$

Eq. (47) is the differential equation for associated Laguerre functions given in general form by

$$xy'' + 2y' + \left[n^* - \frac{k-1}{2} - \frac{x}{4} - \frac{k^2-1}{4x}\right]y = 0, \tag{48}$$

which has a solution possessing an integrable square of the form

$$y = e^{-x/2} x^{(k-1)/2} \frac{d^k}{dx^k} L_{n^*}(x), \tag{49}$$

provided that n^* and k are positive integers. However, n^* does not have to be an integer, it may be any *arbitrary* constant β. Then the corresponding solution is [8]

$$y = e^{-x/2} x^{(k-1)/2} \frac{d^k}{dx^k} L_\beta(x). \tag{50}$$

In the case that n^* is chosen to be an integer in order to obtain the Rydberg formula, $n^* - k \geq 0$ since otherwise $L_{n^*}^k(x)$ of Eq. (49) would vanish. By comparing Eqs. (47) and (48),

$$\frac{k^2 - 1}{4} = \ell(\ell + 1). \tag{51}$$

Thus,

$$k = 2\ell + 1 \tag{52}$$

and

$$n^* - \frac{k-1}{2} = n^* - \ell = \frac{me^2}{\hbar}\left(\frac{\alpha}{2}\right)^{-1}. \tag{53}$$

Substitution of the value of α and solving for W gives

$$W = \frac{1}{2}\frac{me^4}{(n^* - \ell)^2 \hbar^2}. \tag{54}$$

Because of the conditions on n^* and k, the quantity $n^* - \ell$ cannot be zero. It is usually denoted by n and called the principal quantum number. The energy states of the hydrogen atom are

$$W_n = -E_n = \frac{1}{2}\frac{me^4}{n^2\hbar^2} \tag{55}$$

and the corresponding eigenfunctions from Eq. (49) are

$$R_{n,\ell} = c_{n,\ell}\, e^{-x/2} x^\ell L_{n+\ell}^{2\ell+1}(x), \tag{56}$$

where the variable x is defined by

$$x = \alpha r = \frac{\sqrt{8mW}}{\hbar}r = \frac{2me^2}{n\hbar^2}r. \tag{57}$$

In the Bohr theory of the hydrogen atom, the first orbital has a radius in CGS units given by

$$a_0 = \frac{\hbar^2}{me^2} = 0.53 \times 10^{-8} \text{ cm.} \tag{58}$$

Thus, $\alpha = 2/na_0$ and

$$x = \frac{2}{n}\frac{r}{a_0}. \tag{59}$$

The energy states of the hydrogen atom in CGS units in terms of the Bohr radius are given by Eq. (27). From

Eq. (56), $R_{n,\ell}$ for the hydrogen atom ground state is

$$R_{1,0} = c_{1,0}e^{-r/a_0}L_1^1 = 2a_0^{-3/2}e^{-r/a_0}. \tag{60}$$

For this state

$$Y_{00} = cons\tan t = (4\pi)^{-1/2}, \tag{61}$$

when the function is normalized. Thus, the ground state function is

$$\psi_0 = (\pi a_0^3)^{-1/2}e^{-r/a_0}. \tag{62}$$

Immediately further problems arise. Since ℓ must equal zero in the ground state, the predicted angular energy and angular momentum given by Eqs. (36) and (37), respectively, are zero which are experimentally incorrect. In addition, different integer values of ℓ exist in the case of excited electron states. In these cases, the Schrödinger equation solutions (Eqs. (36) and (37)), predict that the excited state rotational energy levels are nondegenerate as a function of the ℓ quantum number even in the absence of an applied magnetic field. Consider the case of the excited state with $n = 2$; $\ell = 1$ compared to the experimentally degenerate state $n = 2$; $\ell = 0$. According to Eq. (37) the difference in angular energy of these two states is 3.4 eV where the expectation radius, $4a_0$, is given by the squared integral of Eq. (70) over space. Thus, the predicted rotational energy in the absence of a magnetic field is over six orders of magnitude of the observed nondegenerate energy (10^{-7}–10^{-6} eV) in the presence of a magnetic field.

Schrödinger realized that his equation was limited. It is not Lorentzian invariant; thus, it violates special relativity. It also does not comply with Maxwell's equations and other first principle laws. Schrödinger sought a resolution of the incompatibility with special relativity for the rest of his life. He was deeply troubled by the physical consequences of his equation and its solutions. His hope was that the resolution would make his equation fully compatible with classical physics and the quantization would arise from first principles.

Quantum mechanics failed to predict the results of the Stern–Gerlach experiment which indicated the need for an additional quantum number. Quantum electrodynamics was proposed by Dirac in 1926 to provide a generalization of quantum mechanics for high energies in conformity with the theory of special relativity and to provide a consistent treatment of the interaction of matter with radiation. From Weisskopf [9], "Dirac's quantum electrodynamics gave a more consistent derivation of the results of the correspondence principle, but it also brought about a number of new and serious difficulties". Quantum electrodynamics: (1) does not explain nonradiation of bound electrons; (2) contains an internal inconsistency with special relativity regarding the classical electron radius — the electron mass corresponding to its electric energy is infinite; (3) it admits solutions of negative rest mass and negative kinetic energy; (4) the interaction of the electron with the predicted zero-point field

fluctuations leads to infinite kinetic energy and infinite electron mass; and (5) Dirac used the unacceptable states of negative mass for the description of the vacuum; yet, infinities still arise.

A physical interpretation of Eq. (28) was sought. Schrödinger interpreted $e\Psi^*(x)\Psi(x)$ as the charge-density or the amount of charge between x and $x + dx$ (Ψ^* is the complex conjugate of Ψ). Presumably, then, he pictured the electron to be spread over large regions of space. Three years after Schrödinger's interpretation, Max Born, who was working with scattering theory, found that this interpretation led to logical difficulties, and he replaced the Schrödinger interpretation with the probability of finding the electron between r, θ, ϕ and $r + dr, \theta + d\theta, \phi + d\phi$ as

$$\int \Psi(r,\theta,\phi)\Psi^*(r,\theta,\phi)\,dr\,d\theta\,d\phi. \tag{63}$$

Born's interpretation is generally accepted. Nonetheless, interpretation of the wave function is a never-ending source of confusion and conflict. Many scientists have solved this problem by conveniently adopting the Schrödinger interpretation for some problems and the Born interpretation for others. This duality allows the electron to be everywhere at one time — yet have no volume. Alternatively, the electron can be viewed as a discrete particle that moves here and there (from $r = 0$ to ∞), and $\Psi\Psi^*$ gives the time average of this motion.

Schrödinger was also troubled by the philosophical consequences of his theory since quantum mechanics leads to certain philosophical interpretations [10] which are not sensible. Some conjure up multitudes of universes including "mind" universes; others require belief in a logic that allows two contradictory statements to be true. The question addressed is whether the universe is determined or influenced by the possibility of our being conscious of it. The meaning of quantum mechanics is debated, but the Copenhagen interpretation is predominant. It asserts that "what we observe is all we can know; any speculation about what a photon, an atom, or even a superconducting quantum interference device (SQUID) really is or what it is doing when we are not looking is just that — speculation" [10]. As shown by Platt [11] in the case of the Stern–Gerlach experiment, "the postulate of quantum measurement (which) asserts that the process of measuring an observable forces the state vector of the system into an eigenvector of that observable, and the value measured will be the eigenvalue of that eigenvector". According to this interpretation every observable exists in a state of superposition of possible states, and observation or the potential for knowledge causes the wavefunction corresponding to the possibilities to collapse into a definite.

According to the quantum mechanical view, a moving particle is regarded as a wave group. To regard a moving particle as a wave group implies that there are fundamental limits to the accuracy with which such "particle" properties as position and momentum can be measured. Quantum predicts that the particle may be located anywhere within its

wave group with a probability $|\Psi|^2$. An isolated wave group is the result of superposing an infinite number of waves with different wavelengths. The narrower the wave group, the greater range of wavelengths involved. A narrow de Broglie wave group thus means a well-defined position (Δx smaller) but a poorly defined wavelength and a large uncertainty Δp in the momentum of the particle the group represents. A wide wave group means a more precise momentum but a less precise position. The infamous Heisenberg uncertainty principle is a formal statement of the standard deviations of properties implicit in the probability model of fundamental particles.

$$\Delta x \Delta p \geqslant \frac{\hbar}{2}. \tag{64}$$

According to the standard interpretation of quantum mechanics, the act of measuring the position or momentum of a quantum mechanical entity collapses the wave–particle duality because the principle forbids both quantities to be simultaneously known with precision.

3. The wave–particle duality is not due to the uncertainty principle

Quantum entities can behave like particles or waves, depending on how they are observed. They can be diffracted and produce interference patterns (wave behavior) when they are allowed to take different paths from some source to a detector — in the usual example, electrons or photons go through two slits and form an interference pattern on the screen behind. On the other hand, with an appropriate detector put along one of the paths (at a slit, say), the quantum entities can be detected at a particular place and time, as if they are point-like particles. But any attempt to determine which path is taken by a quantum object destroys the interference pattern. Richard Feynman described this as the central mystery of quantum physics.

Bohr called this vague principle 'complementary', and explained it in terms of the uncertainty principle, put forward by Werner Heisenberg, his postdoc at the time. In an attempt to persuade Einstein that wave–particle duality is an essential part of quantum mechanics, Bohr constructed models of quantum measurements that showed the futility of trying to determine which path was taken by a quantum object in an interference experiment. As soon as enough information is acquired for this determination, the quantum interferences must vanish, said Bohr, because any act of observing will impart uncontrollable momentum kicks to the quantum object. This is quantified by Heisenberg's uncertainty principle, which relates uncertainty in positional information to uncertainty in momentum — when the position of an entity is constrained, the momentum must be randomized to a certain degree.

More than 60 years after the famous debate between Niels Bohr and Albert Einstein on the nature of quantum

reality, a question central to their debate — the nature of quantum interference — has resurfaced. The usual textbook explanation of wave–particle duality in terms of unavoidable 'measurement disturbances' is experimentally proven incorrect by an experiment reported by Durr et al. [12]. Durr et al. report on the interference fringes produced when a beam of cold atoms is diffracted by standing waves of light. Their interferometer displayed fringes of high contrast — but when they manipulated the electronic state within the atoms with a microwave field according to which path was taken, the fringes disappeared entirely. The interferometer produced a spatial distribution of electronic populations which were observed via fluorescence. The microwave field canceled the spatial distribution of electronic populations. The key to this new experiment was that although the interferences are destroyed, the initially imposed atomic momentum distribution left an envelope pattern (in which the fringes used to reside) at the detector. A careful analysis of the pattern demonstrated that it had not been measurably distorted by a momentum kick of the type invoked by Bohr, and therefore that any locally realistic momentum kicks imparted by the manipulation of the internal atomic state according to the particular path of the atom are too small to be responsible for destroying interference.

4. The correspondence principle does not hold

Recent experimental results also dispel another doctrine of quantum mechanics [13,14]. Bohr proposed a rule of thumb called the correspondence principle [15]. A form of the principle widely repeated in textbooks and lecture halls states that predictions of quantum mechanics and classical physics should match for the most energetic cases.

Bo Gao [13] calculates possible energy states of any chilled, two-atom molecule, such as sodium, that's vibrating and rotating almost to the breaking point. He performs the calculations via quantum mechanical and the so-called semi-classical methods and compares the results. Instead of the results agreeing better for increasingly energetic states. The opposite happens.

5. Classical solution of the Schrodinger equation

Mills has solved and published a solution of a Schrödinger type equation based on first principles [16]. The central feature of this theory is that all particles (atomic-size and macroscopic particles) obey the same physical laws. Whereas Schrödinger postulated a boundary condition: $\Psi \to 0$ as $r \to \infty$, the boundary condition in Mills' theory was derived from Maxwell's equations [17].

For non-radiative states, the current–density function must not possess space-time Fourier components that

are synchronous with waves traveling at the speed of light.

Application of this boundary condition leads to a physical model of particles, atoms, molecules, and, in the final analysis, cosmology. The closed-form mathematical solutions contain fundamental constants *only*, and the calculated values for physical quantities agree with experimental observations. In addition, the theory predicts that Eq. (3), should be replaced by Eq. (65).

$$n = 1, 2, 3, \ldots \quad \text{and} \quad n = \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \ldots \quad . \tag{65}$$

Some revisions to standard quantum theory are implied. Quantum mechanics becomes a real physical description as opposed to a purely mathematical model where the old and the revised versions are interchangeable by a Fourier transform operation [16].

The theories of Bohr, Schrödinger, and presently Mills all give the identical equation for the principal energy levels of the one-electron atom.

$$E_{\text{ele}} = -\frac{Z^2 e^2}{8\pi\varepsilon_0 n^2 a_H} = -\frac{Z^2}{n^2} 2.1786 \times 10^{-18},$$

$$J = -Z^2 \frac{13.598}{n^2} \text{ eV}. \tag{66}$$

The Mills theory solves the two-dimensional wave equation for the charge-density function of the electron. And, the Fourier transform of the charge-density function is a solution of the three-dimensional wave equation in frequency (k, ω) space. Whereas, the Schrödinger equation solutions are three dimensional in spacetime. The energy is given by

$$\int_{-\infty}^{\infty} \psi H \psi \, d\mathbf{v} = E \int_{-\infty}^{\infty} \psi^2 \, d\mathbf{v}, \tag{67}$$

$$\int_{-\infty}^{\infty} \psi^2 \, d\mathbf{v} = 1. \tag{68}$$

Thus,

$$\int_{-\infty}^{\infty} \psi H \psi \, d\mathbf{v} = E. \tag{69}$$

In the case that the potential energy of the Hamiltonian, H, is a constant times the wavenumber, the Schrödinger equation is the well-known Bessel equation. Then with one of the solutions for ψ, Eq. (69) is equivalent to an inverse Fourier transform. According to the duality and scale change properties of Fourier transforms, the energy equation of the present theory and that of quantum mechanics are identical, the energy of a radial Dirac delta function of radius equal to an integer multiple of the radius of the hydrogen atom (Eq. (66)). Bohr obtained the same energy formula by postulating nonradiative states with angular momentum

$$L_z = m\hbar \tag{70}$$

and solving the energy equation classically.

The mathematics for all three theories converge to Eq. (66). However, the physics is quite different. Only the Mills theory is derived from first principles and holds over a scale of spacetime of 45 orders of magnitude: it correctly predicts the nature of the universe from the scale of the quarks to that of the cosmos.

Mills' revisions transform Schrödinger's and Heisenberg's quantum theory into what may be termed a *classical quantum theory*. Physical descriptions flow readily from the theory. For example, in the old quantum theory the spin angular momentum of the electron is called the "intrinsic angular momentum". This term arises because it is difficult to provide a physical interpretation for the electron's spin angular momentum. Quantum Electrodynamics provides somewhat of a physical interpretation by proposing that the "vacuum" contains fluctuating electric and magnetic fields. In contrast, in Mills' theory, spin angular momentum results from the motion of negatively charged mass moving systematically, and the equation for angular momentum, $\mathbf{r} \times \mathbf{p}$, can be applied directly to the wave function (a current–density function) that describes the electron, and quantization is carried by the photon, rather than probability waves of the electron.

6. Fractional quantum energy levels of hydrogen

The nonradiative state of atomic hydrogen which is historically called the "ground state" forms the basis of the boundary condition of Mills' theory [16] to solve the wave equation. Mills further predicts [16] that certain atoms or ions serve as catalysts to release energy from hydrogen to produce an increased binding energy hydrogen atom called a *hydrino atom* having a binding energy of

$$\text{Binding energy} = \frac{13.6 \text{ eV}}{n^2}, \tag{71}$$

where

$$n = \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{p} \tag{72}$$

and p is an integer greater than 1, designated as $H[a_H/p]$ where a_H is the radius of the hydrogen atom. (Although it is purely mathematical, these stable energy levels are also given by both Bohr's and Schrödinger's theories by postulating integer values of the central charge. Justification may be based on notions such virtual particles which are acceptable in other applications of Schrödinger's equation.) Hydrinos are predicted to form by reacting an ordinary hydrogen atom with a catalyst having a net enthalpy of reaction of about

$$m \cdot 27.2 \text{ eV}, \tag{73}$$

where m is an integer. This catalysis releases energy from the hydrogen atom with a commensurate decrease in size of the hydrogen atom, $r_n = na_H$. For example, the catalysis of $H(n = 1)$ to $H(n = 1/2)$ releases 40.8 eV, and the hydrogen radius decreases from a_H to $(1/2)a_H$.

It is taught in textbooks that atomic hydrogen cannot go below the ground state. Atomic hydrogen having an experimental ground state of 13.6 eV can only exist in a vacuum or in isolation, and atomic hydrogen cannot go below this ground state in isolation. However, there is no known composition of matter containing hydrogen in the ground state of 13.6 eV. Atomic hydrogen is radical and is very reactive. It may react to form a hydride ion or compositions of matter. It is a chemical intermediate which may be trapped as many chemical intermediates may be by methods such as isolation or cryogenically. A hydrino atom may be considered a chemical intermediate that may be trapped in vacuum or isolation. A hydrino atom may be very reactive to form a hydride ion or a novel composition of matter. Hydrogen at predicted lower energy levels, hydrino atoms, has been identified in the extreme ultraviolet emission spectrum from interstellar medium. In addition, new compositions of matter containing hydrogen at predicted lower energy levels have recently been observed in the laboratory, which energy levels are achieved using the novel catalysts.

The excited energy states of atomic hydrogen are also given by Eq. (71) except that

$$n = 1, 2, 3, \ldots . \tag{74}$$

The $n = 1$ state is the "ground" state for "pure" photon transitions (the $n = 1$ state can absorb a photon and go to an excited electronic state, but it cannot release a photon and go to a lower energy electronic state). However, an electron transition from the ground state to a lower energy state is possible by a nonradiative energy transfer such as multipole coupling or a resonant collision mechanism. These lower energy states have fractional quantum numbers, $n = 1/\text{integer}$. Processes that occur without photons and that require collisions are common. For example, the exothermic chemical reaction of $H + H$ to form H_2 does not occur with the emission of a photon. Rather, the reaction requires a collision with a third body, M, to remove the bond energy — $H + H + M \rightarrow H_2 + M^*$ [18]. The third body distributes the energy from the exothermic reaction, and the end result is the H_2 molecule and an increase in the temperature of the system. Some commercial phosphors are based on nonradiative energy transfer involving multipole coupling [19]. For example, the strong absorption strength of Sb^{3+} ions along with the efficient nonradiative transfer of excitation from Sb^{3+} to Mn^{2+}, are responsible for the strong manganese luminescence from phosphors containing these ions. Similarly, the $n = 1$ state of hydrogen and the $n = 1/\text{integer}$ states of hydrogen are nonradiative, but a transition between two nonradiative states is possible via a nonradiative energy transfer, say $n = 1$ to $1/2$. In these cases, during the transition the electron couples to another electron transition, electron transfer reaction, or inelastic scattering reaction which can absorb the exact amount of energy that must be removed from the hydrogen atom. Thus, a catalyst provides a net positive enthalpy of reaction of $m \cdot 27.2$ eV (i.e. it absorbs $m \cdot 27.2$ eV where m is an

integer). Certain atoms or ions serve as catalysts which resonantly accept energy from hydrogen atoms and release the energy to the surroundings to effect electronic transitions to fractional quantum energy levels.

Once formed hydrinos have a binding energy given by Eqs. (71) and (72); thus, they may serve as catalysts which provide a net enthalpy of reaction given by Eq. (73). Also, the simultaneous ionization of two hydrogen atoms may provide a net enthalpy given by Eq. (73). Since the surfaces of stars comprise significant amounts of atomic hydrogen, hydrinos may be formed as a source to interstellar space where further transitions may occur.

A number of experimental observations lead to the conclusion that atomic hydrogen can exist in fractional quantum states that are at lower energies than the traditional "ground" ($n = 1$) state. For example, the existence of fractional quantum states of hydrogen atoms explains the spectral observations of the extreme ultraviolet background emission from interstellar space [20], which may characterize dark matter as demonstrated in Table 2. (In these cases, a hydrogen atom in a fractional quantum state, $H(n_i)$, collides, for example, with a $n = 1/2$ hydrogen atom, $H(1/2)$, and the result is an even lower energy hydrogen atom, $H(n_f)$, and $H(1/2)$ is ionized.

$$H(n_i) + H(\tfrac{1}{2}) \rightarrow H(n_f) + H^+ + e^- + \text{photon}. \tag{75}$$
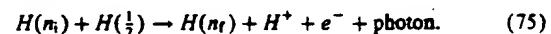
The energy released, as a photon, is the difference between the energies of the initial and final states given by Eqs. (71) and (72) minus the ionization energy of $H(1/2)$, 54.4 eV. The catalysis of an energy state of hydrogen to a lower energy state wherein a different lower energy state atom of hydrogen serves as the catalyst is called disproportionation by Mills [16].

7. Identification of lower energy hydrogen by soft X-rays from dark interstellar medium

The first soft X-ray background was detected and reported [21] about 25 years ago. Quite naturally, it was assumed that these soft X-ray emissions were from ionized atoms within hot gases. In a more recent paper, a grazing incidence spectrometer was designed to measure and record the diffuse extreme ultraviolet background [20]. The instrument was carried aboard a sounding rocket and data were obtained between 80 and 650 Å (data points approximately every 1.5 Å). Here again, the data were interpreted as emissions from hot gases. However, the authors left the door open for some other interpretation with the following statement from their introduction:

It is now generally believed that this diffuse soft X-ray background is produced by a high-temperature component of the interstellar medium. However, evidence of the thermal nature of this emission is indirect in that it is based not on observations of line emission, but on

Table 1
Energies (Eq. (76)) of several fractional-state transitions catalyzed by $H[a_H/2]$

n_i	n_f	ΔE (eV)	λ (Å)
1/2	1/3	13.6	912
1/3	1/4	40.80	303.9
1/4	1/5	68.00	182.4
1/5	1/6	95.20	130.2
1/6	1/7	122.4	101.3
1/7	1/8	149.6	82.9

indirect evidence that no plausible non-thermal mechanism has been suggested which does not conflict with some component of the observational evidence.

The authors also state that "if this interpretation is correct, gas at several temperatures is present". Specifically, emissions were attributed to gases in three ranges: $5.5 < \log T < 5.7$; $\log T = 6$; $6.6 < \log T < 6.8$.

The explanation proposed herein of the observed dark interstellar medium spectrum hinges on the possibility of energy states below the $n = 1$ state, as given by Eqs. (71) and (72). Thus, lower energy transitions of the type

$$\Delta E = \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \times 13.6 \text{ eV} - 54.4 \text{ eV}$$

$$n = 1, \tfrac{1}{2}, \tfrac{1}{3}, \tfrac{1}{4}, \dots \quad \text{and} \quad n_i > n_f. \tag{76}$$

induced by a disproportionation reaction with $H[a_H/2]$ ought to occur. The wavelength is related to ΔE by

$$\lambda \text{ (Å)} = \frac{1.240 \times 10^4}{\Delta E \text{(eV)}}. \tag{77}$$

The energies and wavelengths of several of these proposed transitions are shown in Table 1. Note that the lower energy transitions are in the soft X-ray region.

8. The data and its interpretation

In their analysis of the data, Labov and Bowyer [20] established several tests to separate emission features from the background. There were seven features (peaks) that passed their criteria. The wavelengths and other aspects of these peaks are shown in Table 2. Peaks 2 and 5 were interpreted by Labov and Bowyer as instrumental second-order images of peaks 4 and 7, respectively. Peak 3, the strongest feature, is clearly a helium resonance line: $He(1s^12p^1 \rightarrow 1s^2)$. At issue here, is the interpretation of peaks 1, 4, 6, and 7. It is proposed that peaks 4, 6, and 7 arise from the $1/3 \rightarrow 1/4$, $1/4 \rightarrow 1/5$, and $1/6 \rightarrow 1/7$ hydrogen atoms transitions given by Eq. (76). It is also proposed that peak 1 arises from inelastic helium scattering of peak 4. That is, the $1/3 \rightarrow 1/4$ transition yields a 40.8 eV photon (303.9 Å).

When this photon strikes $He(1s^2)$, 21.2 eV is absorbed in the excitation to $He(1s^12p^1)$. This leaves a 19.6 eV photon (632.6 Å), peak 1. For these four peaks, the agreement between the predicted values (Table 1) and the experimental values (Table 2) is remarkable.

One argument against this new interpretation of the data is that the transition $1/5 \rightarrow 1/6$ is missing — predicted at 130.2 Å by Eqs. (76) and (77). This missing peak cannot be explained into existence, but a reasonable rationale can be provided for why it might be missing from these data. The data obtained by Labov and Bowyer are outstanding when the region of the spectrum, the time allotted for data collection, and the logistics are considered. Nonetheless, it is clear that the signal-to-noise ratio is low and that considerable effort had to be expended to differentiate emission features from the background. This particular peak, $1/5 \rightarrow 1/6$, is likely to be only slightly stronger than the $1/6 \rightarrow 1/7$ peak (the intensities, Table 2, appear to decrease as n decreases), which has low intensity. Labov and Bowyer provided their data (wavelength, count, count error, background, and background error). The counts minus background values for the region of interest, 130.2 ± 5 Å, are shown in Table 3 (the confidence limits for the wavelength of about ± 5 Å are the single-side 1 confidence levels and include both the uncertainties in the fitting procedure and uncertainties in the wavelength calibration). Note that the largest peak (count − background) is at 129.64 Å and has a counts − background = 8.72. The counts − background for the strongest signal of the other hydrino transitions are: $n = 1/3 - 1/4$, 20.05; $n = 1/4 - 1/5$, 11.36; $n = 1/6 - 1/7$, 10.40. Thus, there is fair agreement with the wavelength and the strength of the signal. This, of course, does not mean that there is a peak at 130.2 Å. However, it is not unreasonable to conclude that a spectrum with a better signal-to-noise ratio might uncover the missing peak.

Another, and more important, argument against this new interpretation is the fact that the proposed fractional-quantum-state hydrogen atoms have not been detected before. There are several explanations. Firstly, the transitions to these fractional states must be forbidden or must have very high activation energies — otherwise, all hydrogen atoms would quickly go to these lower energy states (an estimated transition probability, based on the Labov and Bowyer data, is be between 10^{-15} and 10^{-17} s^{-1}). In actuality, a catalyst is required in order to obtain emission. Secondly, the number of hydrogen atoms ($n = 1$), the hydrogen-atom density, and the presence of an active catalyst under any conditions on Earth is exceeding low. The combination of extremely low population and extremely low transition probability makes the detection of these transitions especially difficult. Thirdly, this is a very troublesome region of the electromagnetic spectrum for detection because these wavelengths do not penetrate even millimeters of the atmosphere (i.e. this region is the vacuum ultraviolet which requires windowless spectroscopy at vacuum for detection). Lastly, no one previously has been

Table 2
Emission features of the LaBov and Bowyer spectrum and their interpretation

Peak	λ (Å)	Confidence limit (Å)	Intensity (photons cm^{-2} s^{-1} sr^{-1})	Assignment [20]	Assignment [33]	Predicted λ (Eqs. (76) and (77)) (Å)
1	633.0	−4.7 to +4.7	19,000	O^{++}; $\log T = 5.5$	He scattering of 303.9 line (peak 4)	633.0
2	607.5	−4.9 to +4.9	Second order	Second order of 302.5 line	Second order of 303.9 line	607.8
3	584	−4.5 to +4.5	70,400	He resonance $(1s^12p^1 \rightarrow 1s^2)$	He resonance $(1s^12p^1 \rightarrow 1s^2)$	584
4	302.5	−6.0 to +5.9	2,080	He^+; $(2p^1 \rightarrow 1s^1)$	$n = 1/3–1/4$	303.9
5	200.6	−4.4 to +5.3	Second order	Second order of 101.5 line	Second order of 101.3 line	202.6
6	181.7	−4.6 to +5.1	1030	Fe^{9+} and Fe^{10+}; $\log T = 6$	$n = 1/4–1/5$	182.3
7	101.5	−5.3 to +4.2	790	Fe^{17+} and Fe^{18+}; $\log T = 6.6 – 6.8$	$n = 1/6–1/7$	101.3

Table 3
Data (Labov and Bowyer) near the predicted $1/5 \rightarrow 1/6$ transition (130.2 Å)

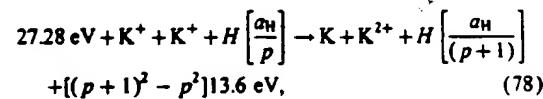| l (Å)	Counts	Background	Counts−background
125.82	26	21.58	4.42
127.10	22	21.32	0.68
128.37	18	19.50	−1.50
129.64	29	20.28	8.72
130.90	18	19.76	−1.76
132.15	20	19.50	0.50
133.41	19	19.50	−0.50
134.65	19	20.80	−1.80

actively searching for these transitions. The Chandra X-ray Observatory is scheduled to perform similar experiments with detection at much better signal to noise than obtained by Labov and Bowyer.

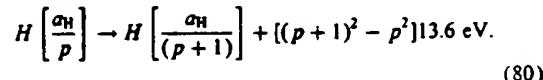## 9. Novel energy states of hydrogen formed by a catalytic reaction

The catalysis of hydrogen involves the nonradiative transfer of energy from atomic hydrogen to a catalyst which may then release the transferred energy by radiative and nonradiative mechanisms. As a consequence of the nonradiative energy transfer, the hydrogen atom becomes unstable and emits further energy until it achieves a lower energy nonradiative state having a principal energy level given by Eqs. (71) and (72).

Potassium ions can also provide a net enthalpy of a multiple of that of the potential energy of the hydrogen atom. The second ionization energy of potassium is 31.63 eV; and K^+ releases 4.34 eV when it is reduced to K. The combination of reactions K^+ to K^{2+} and K^+ to K, then, has a net enthalpy of reaction of 27.28 eV, which is equivalent to

$m = 1$ in Eq. (73).

$$27.28 \text{ eV} + K^+ + K^+ + H\left[\frac{a_H}{p}\right] \rightarrow K + K^{2+} + H\left[\frac{a_H}{(p+1)}\right]$$
$$+ [(p+1)^2 - p^2]13.6 \text{ eV}, \tag{78}$$

$$K + K^{2+} \rightarrow K^+ + K^+ + 27.28 \text{ eV}. \tag{79}$$

The overall reaction is

$$H\left[\frac{a_H}{p}\right] \rightarrow H\left[\frac{a_H}{(p+1)}\right] + [(p+1)^2 - p^2]13.6 \text{ eV}. \tag{80}$$

Typically, the emission of extreme ultraviolet light from hydrogen gas is achieved via a discharge at high voltage, a high-power inductively coupled plasma, or a plasma created and heated to extreme temperatures by RF coupling (e.g. $> 10^6$ K) with confinement provided by a toroidal magnetic field. Intense EUV emission was observed by Mills et al. [22–27] at low temperatures (e.g. $< 10^3$ K) from atomic hydrogen and certain atomized pure elements or certain gaseous ions which ionize at integer multiples of the potential energy of atomic hydrogen. The release of energy from hydrogen as evidenced by the EUV emission must result in a lower energy state of hydrogen. The lower energy hydrogen atom called a hydrino atom by Mills [16] would be expected to demonstrate novel chemistry. The formation of novel compounds based on hydrino atoms would be substantial evidence supporting catalysis of hydrogen as the mechanism of the observed EUV emission. A novel hydride ion called a hydrino hydride ion having extraordinary chemical properties given by Mills [16] is predicted to form by the reaction of an electron with a hydrino atom. Compounds containing hydrino hydride ions have been isolated as products of the reaction of atomic hydrogen with atoms and ions identified as catalysts in the Mills et al. EUV study [16,22–33]. The novel hydride compounds were identified

analytically by techniques such as time of flight secondary ion mass spectroscopy, X-ray photoelectron spectroscopy, and proton nuclear magnetic resonance spectroscopy. For example, the time of flight secondary ion mass spectroscopy showed a large hydride peak in the negative spectrum. The X-ray photoelectron spectrum showed large metal core level shifts due to binding with the hydride as well as novel hydride peaks. The proton nuclear magnetic resonance spectrum showed significantly upfield shifted peaks which corresponded to and identified novel hydride ions.

10. Discussion

The Schrödinger equation gives the observed spontaneously radiative energy levels and the nonradiative state of hydrogen. On this basis alone, it is justified despite its inconsistency with physicals laws and numerous experimental observations such as

- The appropriate eigenvalue must be postulated and the variables of the Laguerre differential equation must be defined as integers in order to obtain the Rydberg formula.
- The Schrödinger equation is not Lorentzian invariant.
- The Schrödinger equation violates first principles including special relativity and Maxwell's equations [16,34].
- The Schrödinger equation gives no basis why excited states are radiative and the 13.6 eV state is stable [16].
- The Schrödinger equation solutions (Eqs. (36) and (37)), predict that the ground state electron has zero angular energy and zero angular momentum, respectively.
- The Schrödinger equation solution (Eq. (37)) predicts that the ionized electron may have infinite angular momentum.
- The Schrödinger equation solutions (Eqs. (36) and (37)), predict that the excited state rotational energy levels are nondegenerate as a function of the ℓ quantum number even in the absence of an applied magnetic field, and the predicted energy is over six orders of magnitude of the observed nondegenerate energy in the presence of a magnetic field. In the absence of a magnetic field, no preferred direction exists. In this case, the ℓ quantum number is a function of the orientation of the atom with respect to an arbitrary coordinate system. Therefore, the nondegeneracy is nonsensical and violates conservation of angular momentum of the photon.
- The Schrödinger equation predicts that each of the functions that corresponds to a highly excited state electron is not integrable and cannot be normalized; thus, each is infinite.
- The Schrödinger equation predicts that the ionized electron is sinusoidal over all space and cannot be normalized; thus, it is infinite.
- The Heisenberg uncertainty principle arises as the standard deviation in the electron probability wave, but experimentally it is not the basis of wave–particle duality.

- The correspondence principle does not hold experimentally.
- The Schrödinger equation does not predict the electron magnetic moment and misses the spin quantum number all together.
- The Schrödinger equation is not a wave equation since it gives the velocity squared proportional to the frequency.
- The Schrödinger equation is not consistent with conservation of energy in an inverse potential field wherein the binding energy is equal to the kinetic energy and the sum of the binding energy and the kinetic energy is equal to the potential energy [2].
- The Schrödinger equation permits the electron to exist in the nucleus which is a state that is physically nonsensical with infinite potential energy and infinite negative kinetic energy.
- The Schrödinger equation interpreted as a probability wave of a point particle cannot explain neutral scattering of electrons from hydrogen [16].
- The Schrödinger equation interpreted as a probability wave of a point particle gives rise to infinite magnetic and electric energy in the corresponding fields of the electron.
- A modification of the Schrödinger equation was developed by Dirac to explain spin which relies on the unfounded notions of negative energy states of the vacuum, virtual particles, and gamma factors.

The success of quantum mechanics can be attributed to (1) the lack of rigor and unlimited tolerance to ad hoc assumptions in violation of physical laws, (2) fantastical experimentally immeasurable corrections such as virtual particles, vacuum polarizations, effective nuclear charge, shielding, ionic character, compactified dimensions, and renormalization, and (3) curve fitting parameters that are justified solely on the basis that they force the theory to match the data. Quantum mechanics is now in a state of crisis with constantly modified versions of matter represented as undetectable minuscule vibrating strings that exist in many unobservable hyperdimensions, that can travel back and forth between undetectable interconnected parallel universes. And, recent data shows that the expansion of the universe is accelerating. This observation has shattered the long held unquestionable doctrine of the origin of the universe as a big bang [35]. It may be time to reconsider the roots of quantum theory, namely the theory of the hydrogen atom. Especially in light of the demonstration that the hydrogen atom can be solved in closed form from first principles, that new chemistry is predicted, and that the predictions have substantial experimental support.

Billions of dollars have been spent to harness the energy of hydrogen through fusion using plasmas created and heated to extreme temperatures by RF coupling (e.g. $> 10^6$ K) with confinement provided by a toroidal magnetic field. Mills et al. [22–27] have demonstrated that energy may be released from hydrogen *using a chemical catalyst* at relatively low temperatures with an apparatus which is of trivial technological complexity compared to a tokomak. And, rather

than producing radioactive waste, the reaction has the potential to *produce compounds* having extraordinary properties [28-33]. The implications are that a vast new energy source and a new field of hydrogen chemistry have been discovered.

References

[1] Beiser A. Concepts of modern physics, 4th ed. New York: McGraw-Hill, 1987. p. 87-117.

[2] Fowles GR. Analytical mechanics, 3rd ed. New York: Holt, Rinehart, and Winston, 1977. p. 154-6.

[3] McQuarrie DA. Quantum chemistry. Mill Valley, CA: University Science Books, 1983. p. 78-9.

[4] Jackson JD. Classical electrodynamics, 2nd ed. New York: Wiley, 1962. p. 84-108.

[5] McQuarrie DA. Quantum chemistry. Mill Valley, CA: University Science Books, 1983. p. 221-4.

[6] Moore W. Schrödinger life and thought. Cambridge: Cambridge University Press, 1989. p. 198.

[7] Fowles GR. Analytical mechanics, 3rd ed. New York: Holt, Rinehart, and Winston, 1977. p. 57-60.

[8] Margenau H, Murphy GM. The mathematics of chemistry and physics. New York: D. Van Nostrand Company, Inc. 1943. p. 77-8.

[9] Weisskopf VF. Rev Mod Phys 1949;21(2):305-15.

[10] Horgan J. Quantum philosophy. Sci Am 1992;267:94-104.

[11] Platt DE. Am J Phys 1992;60(4):306-8.

[12] Durr S, Nonn T, Rempe G. Nature 1998;395:33-7.

[13] Gao B. Phys Rev Lett 1999;83(21):4225-8.

[14] Weiss P. Physics rule of thumb gets thumbs down: Sci News 1999;156:342.

[15] Thomsen DE. Going Bohr's way in physics. Sci News 1/11/86. p. 26.

[16] Mills R. The grand unified theory of classical quantum mechanics, January 1999 ed. Cranbury, NJ: BlackLight Power, Inc. Distributed by Amazon.com.

[17] Haus HA. On the radiation from point charges. Am J Phys 1986;54:1126-9.

[18] Sidgwick NV. The chemical elements and their compounds, vol. I. Oxford: Clarendon Press, 1950. p. 17.

[19] Lamb MD. Luminescence spectroscopy. London: Academic Press, 1978, p. 68.

[20] Labov S, Bowyer S. Spectral observations of the extreme ultraviolet background. Astrophys J 1991;371:810-9.

[21] Bower S, Field G, Mack J. Detection of an anisotrophic soft X-ray background flux. Nature 1968;217:32.

[22] Mills R, Dong J, Lu Y. Observation of extreme ultraviolet hydrogen emission from incandescently heated hydrogen gas with certain catalysts. 1999 Pacific Conference on Chemistry and Spectroscopy and the 35th ACS Western Regional Meeting, Ontario Convention Center, CA. October 6-8, 1999.

[23] Mills R, Dong J, Lu Y. Observation of extreme ultraviolet hydrogen emission from incandescently heated hydrogen gas with certain catalysts. Int J Hydrogen Energy, 2000;25: 919-43.

[24] Mills R. Observation of extreme ultraviolet hydrogen and hydrino emission from hydrogen-KI plasmas produced by a hollow cathode discharge. Int J Hydrogen Energy, submitted.

[25] Mills R. Temporal behavior of light-emission in the visible spectral range from a Ti-K2CO3-H-Cell. Int J Hydrogen Energy, in press.

[26] Mills R, Lu Y, Onuma T. Formation of a hydrogen plasma from an incandescently heated hydrogen-potassium gas mixture and plasma decay upon removal of heater power. Int J Hydrogen Energy, submitted.

[27] Mills R, Nansteel M, Lu Y. Observation of extreme ultraviolet hydrogen emission from incandescently heated hydrogen gas with strontium that produced an optically measured power balance that was 4000 times the control. Int J Hydrogen Energy, in press.

[28] Mills R, Dhandapani B, Greenig N, He J. Synthesis and characterization of potassium iodo hydride. Int J Hydrogen Energy, 2000;25:1185-1203.

[29] Mills R. Novel hydride compound. Int J Hydrogen Energy, in press.

[30] Mills R. Novel hydrogen compounds from a potassium carbonate electrolytic cell. Fusion Technol 2000;37(2): 157-82.

[31] Mills R, He J, Dhandapani B. Novel hydrogen compounds. 1999 Pacific Conference on Chemistry and Spectroscopy and the 35th ACS Western Regional Meeting, Ontario Convention Center, CA, October 6-8, 1999.

[32] Mills R, Dhandapani B, Nansteel M, He J. Synthesis and characterization of novel hydride compounds. Int J Hydrogen Energy, in press.

[33] Mills R. Highly stable novel inorganic hydrides. Int J Inorganic Mater, submitted.

[34] Fuchs CA, Peres A. Quantum theory needs no 'interpretation'. Phys Today 2000;53(3):70-1.

[35] Babcall NA, Ostriker JP, Perlmutter S, Steinhardt PJ. Science 1999;284:1481-8.